The Labour Theory of Value. A Historical-Logical Analysis

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Introduction

The historical-logical analysis is chosen not only because it is the method appropriate for a Marxist study but also to protest against the misuse of the concept by the eminent British Marxist Ronald Meek who pretended that there was a historical reality corresponding to his non-solution of the transformation problem. Ronald Meeks' work is just another example of apologetic orthodox Marxism which is symptomatic for the failure of real existing socialism.

We remind us that the transformation problem comes about as Marx leaves no room for capital in the process of creation of value and surplus value in particular whereas profit as a form of surplus value occurs in proportion to capital. We have to realize that Marx explanation is incorrect and that there cannot be any reality corresponding to his thesis.

On the other hand once a proper historical-logical analysis is conducted the real law of value is unveiled and the Marxian theory is corrected and perfected.

We shall begin with some logical considerations. The basic proposition of the labour theory of value is the proportionality of value and labour expended in the production of commodities. When px is the money value of some commodity (**p** price, **x** quantity) then labour embodied $\mathbf{L_e}$ multiplied by the wage rate must be equal to this monetary value.

$$\mathbf{px = w\ L_e}$$

The difficulty is to gain a proper understanding of the variables involved.

Marx has properly argued that the worker does not get paid the work he does but only what he needs to reproduce his labour power that is the costs of labour. The value of the wages paid to the workers considered Marx as capital, more precisely as variable capital v which together with the labour embodied in the means of production, the constant capital c as well as the surplus value m makes up the total labour value of the commodity.

$$\mathbf{L_e = v + c + m}$$

As already remarked there is no relation between surplus value and constant capital in Marx. Marx assumes as a matter of logical necessity that there is a unique rate of exploitation, the ratio of surplus value to variable capital **s=m/v**. This presupposes a competition amongst workers not only for the wage rate but also for the rate of exploitation. We shall come back to this point later. Marx has constructed his theory on the basis of Ricardo's work and has inherited Ricardo's shortcomings. To resolve the puzzle we shall instead investigate the reality of the capitalistic production process and by that reveal the correct value relations as they are manifested in practice.

Part I. Labour Value and Perfect Competition

The determination of socially necessary labour

As a starting point we take the profit maximizing producer who applies his capital under the conditions of perfect competition. It is this environment which is also taken by Marx for granted as long as he investigates the pure value relations. It must be the capitalistic producer upon whom rests the task to determine the socially necessary labour for the production of a commodity as it is him who decides upon the use of resources. By profitability considerations the capitalistic producer chooses to combine his factors of production in such a way that the value of the marginal product of each factor equals the price of its service. For the case of labour this means that the capitalist adjusts the amount of labour in relation to capital so that the wage rate equals the marginal productivity of labour **δx/δL** multiplied with the price of the product.

We have now a first expression for the wage rate, our variable **w**.

$$w = \frac{\delta x}{\delta L} p$$

But we have not received something as expected. The wage rate should be a ratio of some things like money per labour. Instead we have received a product. In all modern microeconomic textbooks this is how the wage rate is represented under conditions of perfect competition. Fortunately it is easy to overcome the problem. In general, production functions from which the marginal productivity is derived are invertible and so we may take the inverse of the marginal productivity instead in order to get our ratio, more appropriate for the wage rate. **w** is equal to the price in relation to the inverse of the marginal productivity of labour.

$$w = \frac{p}{\delta L / \delta x}$$

In fact it is almost impossible to find the expression **δL/δx** anywhere in economic books, that is it is almost impossible. There is at least one exception and that is no one else than one of the three marginal (counter)-revolutionaries. It is William Stanley Jevons in his Theory of Political Economy (4^{th} ed. p.177).

It seems to be most appropriate to call **δL/δx** marginal labour value. Now it remains to find a proper expression for our variable $\mathbf{L_e}$. As a matter of mathematical necessity the amount of embodied labour must be expressed as $\frac{\delta L}{\delta x} x$, the product of marginal labour value and the quantity of the commodity **x**. Our value equation becomes now

$$px = wL_e = \frac{p}{\delta L / \delta x} \frac{\delta L}{\delta x} x$$

This is a rather remarkable result. We have established that in a capitalistic economy the law of value manifests itself via the profit maximizing behaviour of the capitalists. Indeed marginal analysis allows identifying the labour content of a commodity without knowing the complexities of the production processes which have lead to the creation of the means of production and the labour used up in the production process. In fact the behaviour of the capitalist determines what Marx called the socially necessary labour for the production of the commodity in question.

We should observe also that marginal analysis has overcome the classical problem of labour commanded (defined as monetary value divided by the wage rate) and labour embodied. A value analysis based on some average labour hour of some average intensity does not allow a solution but marginal analysis is capable of taking care of differences in capital labour ratios. It is not only this problem which is resolved by marginal analysis. Another difficulty arises in the context of the classical framework by changes of the rate of interest and its effects on value. Now the reason for this becomes obvious. The capitalist changes the optimal factor proportions so that the rate of profit equals the rate of interest. But when the factor proportions change these change also the marginal productivity of labour and its inverse, marginal labour value. It is important to realise that the factor proportions determine both, the rate of profit as well as the marginal productivity of labour and the wage rate. The so called factor-price frontier represents well the antagonistic character of capitalistic production. Furthermore Marx famous law of the tendency of the rate of profit to fall becomes undeniable as it is nothing else but the law of the diminishing marginal productivity of capital.

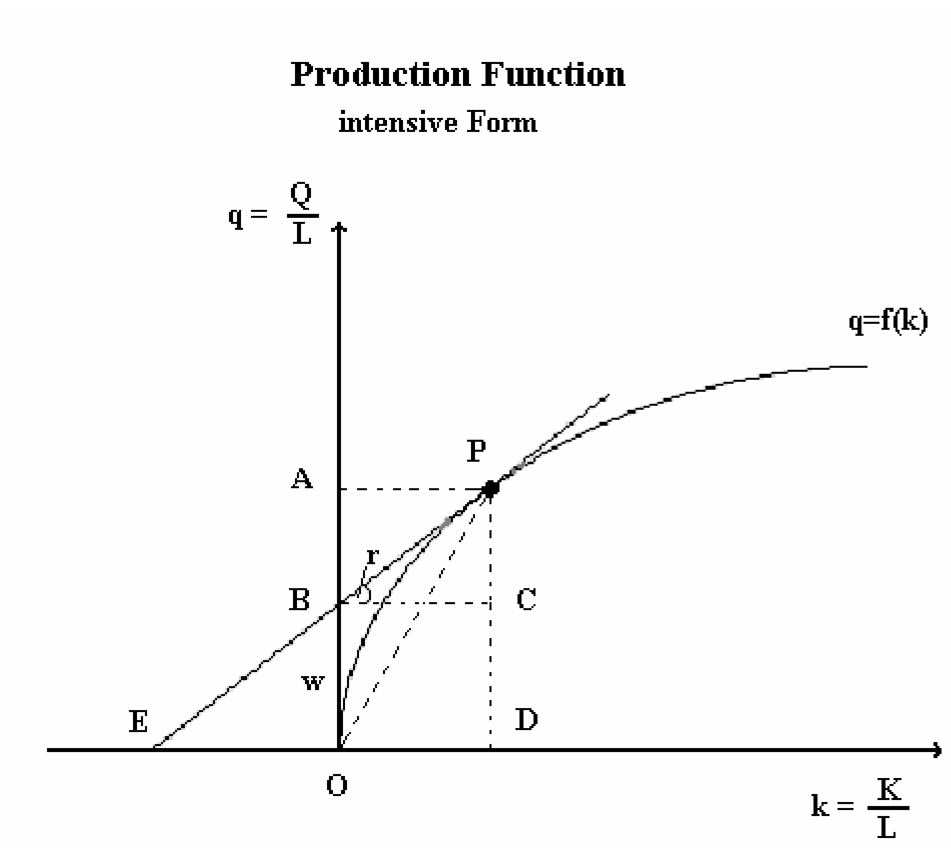


Figure 1

Marginal analysis presupposes a substitutional production function as depicted in Figure 1. In its intensive form the function shows output per labour **q** as a function of the capital labour ratio **k** which is equal to the distance **OD.** Its slope represents the marginal productivity of capital which equals the rate of profit r as in point **P** for example. The shape of the production function is concave to the origin which shows the diminishing marginal productivity of capital (falling rate of profit). The distance **AB** measures surplus per labour and the distance **BO** the wage rate **w**. The factor price ratio is measured by the distance **EO** and the capital output ratio is measured by the ratio **OD/PD**.

The relationship between marginal labour values and labour hours

One might argue that marginal labour value is not an appropriate measure of value because it is expression of some ratio of factor proportions only and labour value must be expressed as some labour time. Labour time is life time. A closer investigation resolves also this issue. The relation of marginal labour value and labour hours can best be described with the help of the production elasticity of labour **a** which is the percentage change of output **δx/x** related to the percentage change of labour hours as input **δL/L** or equivalently as the ratio of marginal labour productivity **δx/δL** to average labour productivity **x/L**.

$$a = \frac{\delta x}{\delta L}\frac{L}{x}$$

L/x is called the labour coefficient. Now it is clear that marginal labour value equals the ratio of the labour coefficient to the production elasticity of labour. Or alternatively one might express marginal labour value as the labour coefficient weighted by the inverse of the production elasticity of labour

$$\frac{\delta L}{\delta x} = \frac{1}{a}\frac{L}{x}$$

Again another presentation is

$$\frac{\delta L}{\delta x}x = \frac{1}{a}L$$

This represents labour value as equal to labour inputs weighted by the inverse of the production elasticity of labour.

We may further clarify the meaning of marginal labour value by pointing out that it is equal to labour hours if the production process exhibits a production elasticity of labour of unity (**a = 1**) which is the case for a production process which does not use any capital at all and which has constant returns to scale. In this sense we may say that marginal labour value represents simple labour, unassisted by capital whereas labour combined with capital yields more labour value, taken that the value of the production elasticity of labour is less than unity. We may observe at this point that surplus value is indeed a function of the capital labour ratio and the rate of surplus value differs between employments. This implies that workers compete only for wages. The question of the rate of exploitation and the capital intensity of production and its relation to the wage rate remains to be analysed in the context of the theory of the structure of earnings.

Relative prices in terms of marginal labour value and labour inputs

The most interesting aspect of the labour theory of value is its ability to determine relative prices.

From our definition of the wage rate follows

$$p_i = w \frac{\delta L}{\delta x_i}$$

As w is unique in the economy under perfect competition relative prices are equal to the ratio of marginal labour values:

$$\frac{p_1}{p_2} = \frac{\delta L / \delta x_1}{\delta L / \delta x_2}$$

From microeconomic textbooks one is more familiar with the expression

$$\frac{p_1}{p_2} = \frac{\delta x_2 / \delta L}{\delta x_1 / \delta L}$$

without finding any explanation why it is the reciprocal of the marginal productivities which matters.

But we are now also capable to determine relative prices in terms of direct labour inputs as

$$\frac{p_1}{p_2} = \frac{\delta L / \delta x_1}{\delta L / \delta x_2} = \frac{a_2}{a_1} \frac{L_1 / x_1}{L_2 / x_2}$$

Under conditions of perfect competition relative prices are equal to the ratio of labour coefficients weighted with the inverse of the ratio of the production elasticities of labour.

The determination of prices

Marginal analysis brings with it a problem classical and orthodox Marxist analyses do not know. In the classical Marxist analysis prices are determined by the amount of labour hours needed to produce the commodities. But as we have seen this leads to inconsistencies in the treatment of capital. The existence of continuous production functions poses the question of what factor proportions are being used. Once the factor proportions are being determined the prices are determined. The factor proportions themselves depend on the distributional variables, the wage rate and the rate of interest. But what does determine these. The Marxists' answer is clear: the class struggle. And this is surely one aspect but not the only determining factor. The issue can be illustrated by means of the production possibility frontier (PPF). This frontier is the locus of all efficient production possibilities of an economy. Below the frontier production is inefficient whereas the space above the frontier is not attainable because of lack of resources. One can show that for substitutional production functions of a neo-classical type the PPF is convex to the origin. The figure 2 is demonstrating the PPF for 2 commodities.

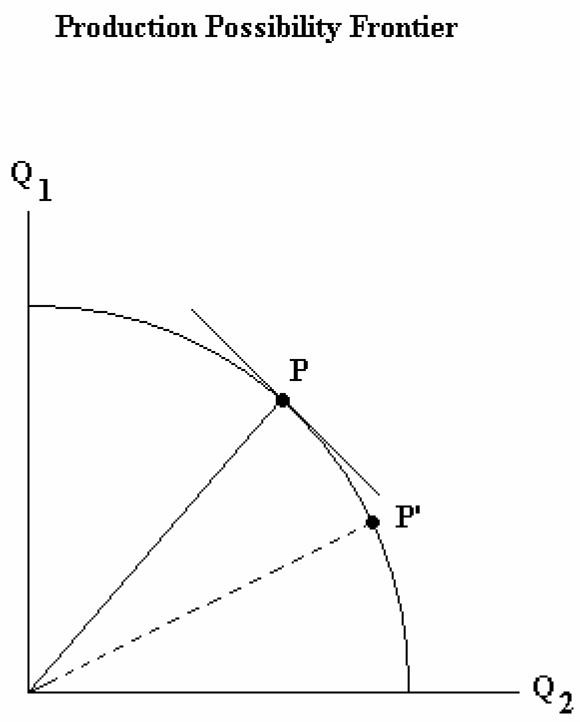


Figure 2

One should observe that each point on the Production Possibility Frontier is characterised by a different ratio of prices. The tangent to a point yields the relative prices. This is so because each point also represents a different factor-price ratio w/r and different factor proportions. To maintain that the factor price ratio is determined via the class struggle would mean in fact that the class struggle would determine also the point on the PPF that is the composition of output.

But the composition of output is clearly a function of demand. On the other hand the demand does depend on the distributional variables, the wage rate as well as the rate of interest. Only when demand is such that it coincides with the conditions of production than the economy is iin equilibrium. Otherwise there would be conflicts which would lead to processes which force production to be below the PPF. In fact this is the core of the problems of macro-economics.

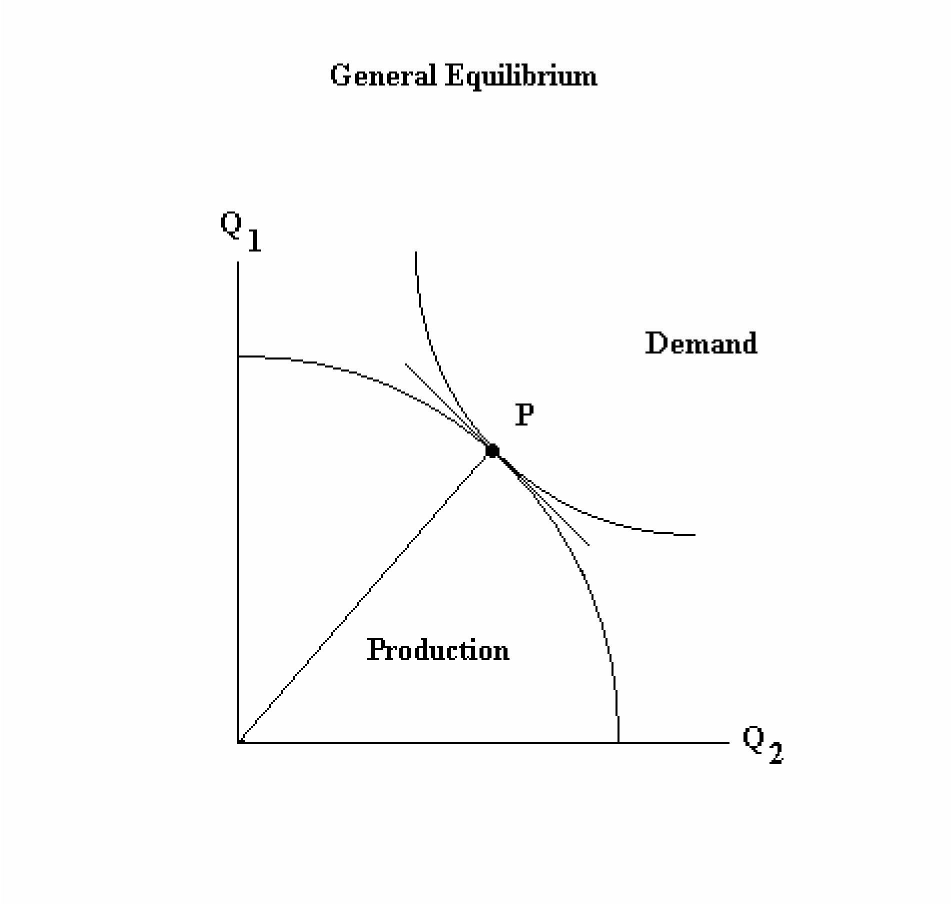


Figure 3

The distribution of the value of labour amongst the 'agents' of production

Adam Smith maintained that on the one hand the value of commodities consists of labour. On the other hand this value is distributed amongst the labourer, the landlord and the capitalist. But Adam Smith was not able to explain this properly.

We have now the theoretical basis for the explanation.

The labour value of a commodity is its marginal labour value **$\delta L/\delta x$** multiplied with the quantity **x**. On the other hand the owners of the factors of production are capable due to competition to acquire the value of the marginal product of their factors of production which under conditions of constant returns to scale is equal to the labour value.

It is

$$\frac{\delta L}{\delta x}x = \frac{\delta L}{\delta x}\frac{\delta x}{\delta q_1}q_1 + \frac{\delta L}{\delta x}\frac{\delta x}{\delta q_2}q_2 + ... + \frac{\delta L}{\delta x}\frac{\delta x}{\delta L}L$$

v_i, i = 1, 2 … n are the quantities of the means of production (the capital)

This simplifies to

$$\frac{\delta L}{\delta x}x = \frac{\delta L}{\delta x}\frac{\delta x}{\delta q_1}q_1 + \frac{\delta L}{\delta x}\frac{\delta x}{\delta q_2}q_2 + ... + L$$

The values $\frac{\delta L}{\delta x}\frac{\delta x}{\delta q_1}q_1 + \frac{\delta L}{\delta x}\frac{\delta x}{\delta q_2}q_2 ...$ represent the constant capital plus surplus value.

We have shown here that Adam Smith thesis that the wealth of nations consists of labour does not only apply to the times when there where no nations yet but also to the modern times of capitalism.

The same results obtained by linear algebra

It remains to be seen that the analysis of the interrelations of the production processes as they are characteristic of modern production yield the same results as marginal analysis.

In order to do so we take an equilibrium situation and we define the production coefficients of this equilibrium situation for all factors of production. The production coefficients are organised as is convenient in input-output analysis as a square matrix. Each factor of production (except labour) is produced and serves as well for the production of all factors of production. The columns of the matrix represent the different inputs of the production of a commodity in one production process and the rows represent the use of the commodity in the different production processes. The production system can be described as a system of equations as

$$\mathbf{p = (1+r)\, pA + wL}$$

A is the matrix of technical coefficients **L** the row vector of labour coefficients, **p** the row vector of prices, **r** the rate of profit.

The equation means that the monetary value of the commodities is the sum of the value of the means of production plus profits plus the monetary value of the labour inputs.

A rearrangement of the terms yields

$$\mathbf{p\, [I - (1+r)A] = wL}$$

We assume now that the matrix [I – (1+r)A] is a non-singular matrix (the determinant is not zero) so that we can derive its inverse and we obtain the solution for the vector of prices as

$$\mathbf{p = wL\, [I - (1+r)\, A]^{-1}}$$

We reinterpret now our value equation in terms of vectors.

When we interpret **p** as a row vector of prices the labour theory of value holds if the row vector

$$\mathbf{L\ [I - (1+r)\ A]^{-1}}$$

is equal to the vector of labour embodied. In Sraffian economics this vector is known as the vector of quantities of dated labour. This is because it can be shown (see Pasinetti 1977) that the vector is equal to a power series expansion.

$$\mathbf{L\ [I - (1+r)\ A]^{-1} = L + L(1+r)A + L(1+r)^2A^2 + L(1+r)^3A^3 + \ldots}$$

We know that the marginal labour value $\boldsymbol{\delta L/\delta x}$ is an appropriate presentation of labour embodied. It remains to be shown that the vector $\mathbf{L\ [I - (1+r)\ A]^{-1}}$ is equal to the vector of marginal labour values

We consider that the economy allocates labour efficiently in all its lines of production. This means that in all sectors the wage must be equal to the value of the marginal product or as we have shown to the ratio of price to marginal labour value. We express this by taking a diagonal matrix **W** in which on the major diagonal all elements represent these ratios of prices to marginal labour values and we write:

$$\mathbf{p = L\ [I - (1+r)\ A]^{-1}\ W}$$

Now it is obvious that our row vector must be equal to the row vector of marginal labour values

$$\mathbf{L\ [I - (1+r)\ A]^{-1} = [\delta L/\delta x_1, \delta L/\delta x_2, \delta L/\delta x_3, \ldots]}$$

This is indeed true if all the other factors of production are also efficiently allocated.

Marginal analysis and Marxian analysis

We have presented here an analysis where marginal labour values are proportional to prices and have with this a consistent theory of value and prices which is lacking in the original Marxist analysis. Furthermore we have confirmed the law of the tendency of the rate of profit to fall as it is nothing else but the law of diminishing marginal productivity of capital which is generally accepted by economists. More difficult is the Marxian proposition of a general decline of wages. Under competitive conditions a decline of the rate of profit entails a rise of the wage rate. This follows from the antagonistic relationship of wages and profits. But we are still considering only the state of perfect competition. Things are different under imperfect competition as in monopoly capitalism.

An important aspect of marginal theory is that the determination of prices depends not only on production conditions but also on demand conditions. This is because with production functions of a continuous form there are unlimited possibilities of factor combinations and a choice has to be made between them.The narrowing of the explanation of history via the conditions of production alone can no longer be maintained. But equally important is that we have the wage rate as well as the rate of profit as functions of the proportions of factor inputs and independent of the prices of commodities. In the context of historical materialism this means that the conditions governing these relations determine the development of the economy and these relations are indeed at the heart of the history of a capitalistic society. It is in this context where the Marxian reproduction schemes as first examples of a 2 sector model of the economy acquire particular interest.

Finally we may realise that the Marxian analysis of the cost of production prices has found a very good development in Sraffian analysis which is at its best when combined with the marginal analysis of labour value. Indeed it is this combination of methods of analysis which shows the validity of the labour theory of value. Marginal analysis is somewhat symmetrical in its treatment of factors of production. One could try to construct relative price being expressed as ratios of marginal productivities of some other "basic" commodity serving as a factor of production. But this symmetry can not be used to claim that the value of commodities is due to quantities of such a basic commodity used in its production because the value added by labour can not be reduced to some quantity of a basic commodity. On the other hand marginal labour value and Sraffian analysis assume that labour is the only primary factor of production.

Part II. Labour Values and Imperfect Competition

One may criticise the historical-logical analysis as conducted above as there is historically no perfect competition. But it is also clear that no one expects the labour theory of value to hold under conditions of imperfect competition. Nevertheless as this is closer to reality and therefore more useful we shall investigate the case. We adjust the model so that the producer faces a normal downward sloping demand curve. When he increases output the price will fall. But we retain constant factor prices and constant returns to scale.

In this situation the producer maximizes his profit when he equalizes the value of the marginal revenue product with the prices for the factor services. For labour this is

$$w = \frac{d(px)}{dx} \delta x / \delta L$$

This is the expression as we know it from microeconomic theory. Now we express it in terms of marginal labour value and we get

$$w = \frac{d(px)}{dx} \delta x / \delta L = \frac{\delta(px)/\delta x}{\delta L/\delta x}$$

We see that the numerator has changed. The increase in revenue is not any more simply the product of price times quantity but it is the partial derivative of the revenue with respect to quantity. This is because the price will change as well as the quantity.

$$\frac{\delta(px)}{\delta x} = p + \frac{\delta p}{\delta x}x = p(1 + \frac{\delta p}{\delta x}\frac{p}{x}) = p(1 + \frac{1}{e})$$

Here e is the price elasticity of demand. This elasticity is negative and so the whole expression in brackets is < 1. Compared with perfect competition it turns out that the value of marginal revenue is smaller than the price.

$$p(1 + \frac{1}{e}) < p$$

Now we consider labour commanded (px/w). Under imperfect competition the wage rate is equal to the value of the marginal revenue product or equivalently is equal to the ratio of marginal revenue to marginal labour value.

$$w = \frac{p(1 + \frac{1}{e})}{\delta L / \delta x}$$

From this follows that labour commanded $\mathbf{L_c}$ is

$$L_c = \frac{px}{w} = \frac{px}{p(1 + \frac{1}{e})}\frac{\delta L}{\delta x} = \frac{1}{(1 + \frac{1}{e})}\frac{\delta L}{\delta x}x$$

We see that labour commanded $\mathbf{L_c}$ is different from labour embodied $\frac{\delta L}{\delta x}x$ by the factor

$$\frac{1}{(1 + \frac{1}{e})}$$

which expresses the amount of extra profits gained by the monopolist.This profit is a function of the price elasticity of demand. It is a gain if 1/(1+1/e) > 1 or e < -1. The closer e to -1, the greater the gain.

We may observe that the term is not defined for the elasticity e = -1. But this point on the demand curve is important as it is the point of maximum revenue. But the point of maximum revenue is only also the point of maximum profit if marginal costs are equal to 0. This is generally not the case.

The proof is straight forward. Revenue R is px. Maximum revenue is at the point where the marginal revenue with respect to x is 0 and the second derivative of revenue with respect to x is negative (which can be taken for granted).

R = px

Maximum Revenue where $\frac{\delta R}{\delta x} = p(1 + 1/e) = 0\,!$

From this follows $1 + 1/e = 0$

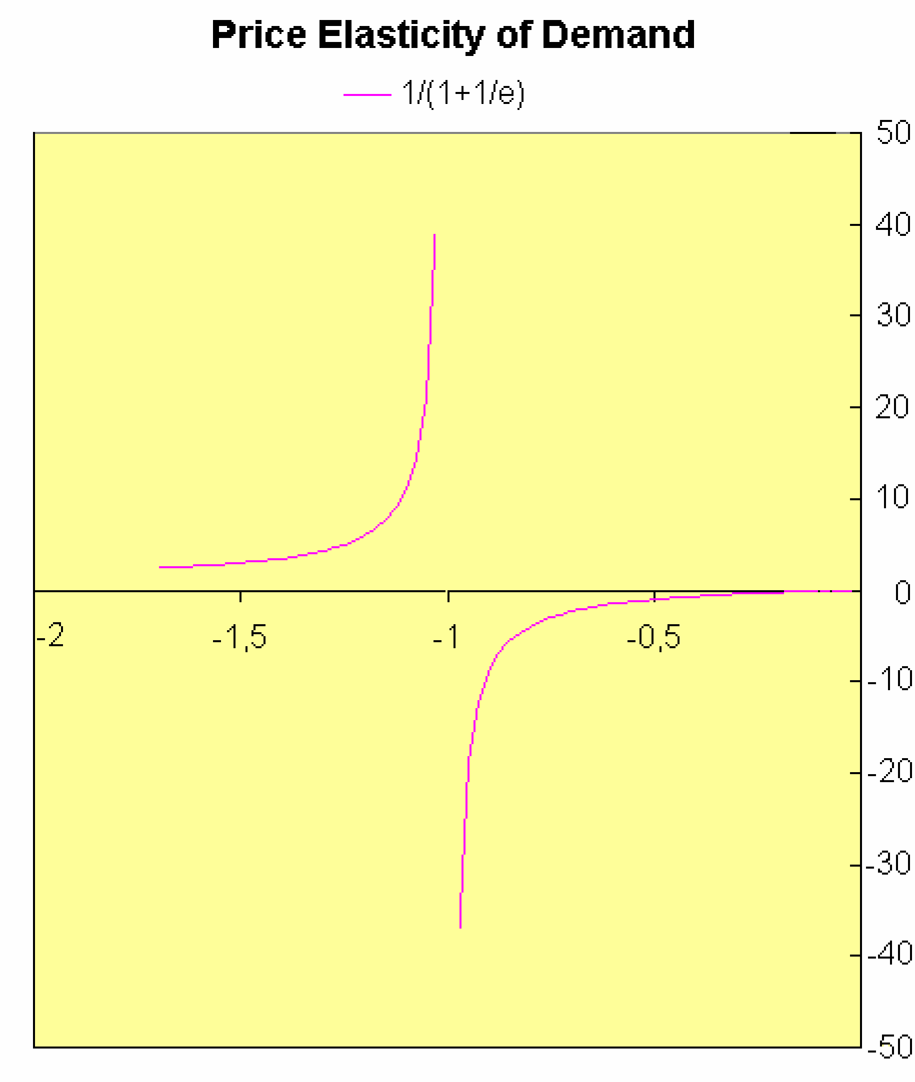
and therefore $e = -1$

The point of the demand curve where **e = -1** is the profit maximizing output of the monopolist if his marginal costs are zero. Usually the output is less and price is higher for maximum profits. Compared with perfect competition this means that price is higher and employment is less.

Another interesting question is relative prices. Now we can establish relative prices for imperfect markets. From our expression of the wage rate we obtain

$$w = \frac{p_1 (1 + \frac{1}{e_1})}{\delta L / \delta x_1} = \frac{p_2 (1 + \frac{1}{e_2})}{\delta L / \delta x_2}$$

$$\frac{p_1}{p_2} = \frac{(1 + \frac{1}{e_2})}{(1 + \frac{1}{e_1})} \frac{\delta L / \delta x_1}{\delta L / \delta x_2}$$



Finally we express relative prices in terms of direct labour inputs as

$$\frac{p_1}{p_2} = \frac{(1+\frac{1}{e_2})}{(1+\frac{1}{e_1})}\frac{\delta L/\delta x_1}{\delta L/\delta x_2} = \frac{(1+\frac{1}{e_2})a_2}{(1+\frac{1}{e_1})a_1}\frac{L_1/x_1}{L_2/x_2}$$

For a given labour input, the price of the product is higher the lower the production elasticity of labour and the closer the price elasticity of demand to -1.

The ratio of extra profits to total revenue

From the result of factor prices being equal to the marginal revenue product one may calculate the ratio of extra profits to total revenue.

Total revenue is equal to total costs plus extra profits EP.

$$px = \left[p(1+\frac{1}{e})\frac{\delta x}{\delta L}L + p(1+\frac{1}{e})\frac{\delta x}{\delta K}K \right] + EP$$

This simplifies to

$$px = (1+\frac{1}{e})p\left[\frac{\delta x}{\delta L}L + \frac{\delta x}{\delta K}K \right] + EP$$

And from this follows under the assumption of constant returns to scale

$$px = (1+\frac{1}{e})px + EP$$

From this we get

$$\frac{EP}{px} = -\frac{1}{e}$$

At the profit maximizing output level the ratio of extra profit to total revenue is equal to the negative inverse of the price elasticity of demand at that output level.

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